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Note on Le Verrier's Tables of Saturn. By A. M. W. Downing, D.Sc., F.R.S.

My attention has been called to the rapidly-increasing discordance between the places of Saturn derived from Le Verrier's tables and those derived from the tables of Hill, as shown by the latest available data, *i.e.* the Ephemerides for 1909.

The following statement gives the corrections to Le Verrier's geocentric places, near the time of opposition in 1909, as given in the *Connaissance des Temps* for that year, deduced from their comparison with the corresponding places given in the *Nautical Almanac* :—

Day 1909.	R.A.	Dec.
	s	″
Oct. 2	– 1·09	– 6·8
10	– 1·10	– 6·9
18	– 1·08	– 6·8

The corresponding corrections to the heliocentric longitudes are :—

Day 1909.	Longitude
	″
Oct. 2	– 16·0
10	– 15·9
18	– 15·9

It will be remembered that M. Gaillot has published "Tables rectifiées du mouvement de Saturne" in tome xxiv. of the *Annales de l'Observatoire de Paris*, and it is of interest to exhibit the resulting corrections to Le Verrier's heliocentric longitudes according to Hill, and according to Gaillot, at convenient epochs, as follows :—

Epoch.	Hill.	Gaillot.
	″	″
1894 March 6	– 3·8	...
1901 Jan. 1	– 3·5	– 4·9
1909 Oct. 10	– 15·9	– 16·2

The observed mean correction to Le Verrier's longitudes of Saturn, derived from the Greenwich Observations of 1900, is – 4″, whilst the observed mean correction to Hill's longitudes, derived from the Greenwich Observations of the following year (the first year in which Hill's tables were used in the *Nautical Almanac*), is about half a second in the same sense; thus demonstrating the superior accuracy of Hill and Gaillot at the commencement of the century. But it is important to note the rapid increase in the discordance between Le Verrier and the other two authorities since that time.

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Computation of Secular Perturbations. By R. T. A. Innes.

I.

It is well known that the Lagrangian equations for the variation of a planet's elements (the planet being considered of infinitesimal dimensions) are exact. It is in the integration of these equations that difficulties arise. To avoid these difficulties several assumptions are made, such as that the planets can be considered in pairs (*i.e.* the problem of three bodies) and do not react on each other; that the elements on the right-hand side of the equations are constant, and that the perturbative function can be developed in a converging series. When we are concerned with the mutual perturbations of the eight major planets, the above assumptions are sufficiently good; the errors introduced by the first two assumptions are eliminated later by taking into account the powers of the masses higher than the first, the third assumption is nearly justified by the smallness of the eccentricities and mutual inclinations of the eight major planets. In cases of difficulty, the Lagrangian equations are abandoned and the perturbations found by other processes; thus Hill found that the Lagrangian method as used by Le Verrier for the theory of the motion of Jupiter and Saturn was insufficient, so that in his theory he adopted the processes developed by Hansen.

Gauss showed that the secular part of the perturbations (to the first power of the masses) could be found without using a development of the perturbative function. The rationale of Gauss's method is as follows:—Let us imagine that the perturbative function is developed in a series as follows,

$$P_0 = P_1 + P_s \sin M + P_c \cos M + P_{2s} \sin 2M + \text{etc.}$$

where the coefficients P contain, besides certain quantities depending on the elements of the disturbed planet, the elements and position of the disturbing body. Multiplying each side by dM and integrating around the circumference gives

$$\frac{1}{2\pi}\int_0^{2\pi} P_0 dM = P_1$$

Then writing $P_1 = p_1 + p_s \sin M_1 + p_c \cos M_1 + \text{etc.}$, where the coefficients p depend only on the elements of both planets and do not contain the time explicitly, a second integration gives

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} P_0 dM dM_1 = p_1$$

It is on the quantity p_1 that the secular perturbations depend. In the general case it is impossible to find this quantity by algebraical expansions. Gauss proved that one of these integrations could be

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